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Long-Range Planning of Power Distribution Systems : Secondary Networks

by

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ABSTRACT

This paper presents an optimization-based methodology for low-voltage distribution network design. The system is composed of three mathematical models: distribution tranformer location, secondary feeder routing and transformer sizing, and primary-transformer interconnection. A heuristic solution approach is used which tests the actual total installation cost for an increasing number of transformers until the point of minimum cost solution, that meets the forecast demand and problem constraints, can be identified. Results are compared with those obtained by using an empirical approach.

Keywords: Power distribution planning, low-voltage distribution system design, network synthesis.

RÉSUMÉ

Ce travail présente une méthodologie d'optimisation pour le projet d'un réseau de distribution d'énergie électrique à basse tension. Le système est composé de trois modèles mathématiques: un modèle de localisation de transformateurs pour la distribution, un modèle de dimensionnement de transformateurs et de localisation des conducteurs, et un modèle de connexion de transformateurs primaires. On utilise une approche heuristique pour calculer le meilleur coût d'installation pour un nombre croissant de transformateurs, qui satisfait la prévision de la demande et les autres contraintes du problème. On compare les résultats avec ceux obtenus par une approche empirique.

Mots-clés: planification de la distribution d'énergie électrique, projet de systèmes de distribution à basse tension, synthèse de réseaux.

1. INTRODUCTION

This paper focuses on a new methodological approach for secondary distribution network design. Many factors reveal the importance of this part of the electric power system, among them :1) The secondary system direct link with the consumers. 2) It operates on a low-voltage level. 3) The costs involved in its construction, operation and maintenance are very high. In secondary network design we are basically concerned with defining the number, the location and sizing of distribution transformers, the feeder routing and sizing, and the primary-transformer interconnection. These questions treated as a whole produce a highly combinatorial problem which is very difficult to solve. Hence, the development of computational tools capable of supporting decision-making are of great interest.

There are few papers that deal with the secondary network design problem via a mathematical programming approach, especially when compared with the large number of articles devoted to primary network planning [1]. Even though the aims of these two network problems are similar, there are inherent peculiarities in the secondary system which demand different models and solution techniques from those used for the primary system [2].

An interesting technique for secondary system design has been presented by Backlund and Bubenko [3]. An initial secondary network configuration is obtained by a minimal spanning tree algorithm (MST), in which every transformer candidate node is considered. The method then attemps to improve the MST solution by exchanging edges of the tree. This approach has some limitations. The initial solution obtained

by the MST algorithm disregards crucial aspects of the problem such as the cost of losses and voltage drop constraints. This may lead to some unacceptable solutions. Furthermore, the improvement technique depends on the node chosen to initiate the edge exchange procedure. The running times to attain improved solutions are likely to be rather long.

In this paper, the design of secondary networks is divided into three hierarchical subproblems: 1) transformer location; 2) secondary feeder routing and transformer sizing; 3) primary feeder routing. The computational systems developed for solving these subproblems constitute an automated methodology that can be used in areas where there is no existing secondary network. In areas where the network is already present the problem refers to the improvement or expansion of the network. The author's current research focuses on this adaptation.

2. PROBLEM DESCRIPTION

The objective of a secondary distribution design system is to construct a network that meets the consumers' forecast demand in a secure, reliable and economical manner. To achieve this aim it attempts to minimize the investment cost and the cost of losses, subject to problem constraints which include the equipment capacity, the maximum voltage drop and the network radial configuration.

The low-voltage operation can promote high losses and voltage drops. Furthermore, typically there is no corrective equipment installed in this part of the distribution system, which means that this is the actual voltage level delivered to the consumer. These aspects draw special attention to the voltage drop limits.

To briefly describe the secondary network design process, take a circumscribed area, for instance, a new housing development. Suppose that the poles are already located and the possible links between then are also determined. Each pole is assigned a demand referring to the consumers served. Also the possible primary voltage supply points, which will connect the primary network to the secondary network, are selected.

The poles in this area represents the places where transformers can be installed. Secondary distribution transformers are available in several sizes and their cost show a heavy economy of scale. In consequence, it is difficult to select the optimal number of transformers to be installed. Indeed, there is a trade off defined by two possible actions: 1) Using a few large transformers and then having to install large and lengthy conductors which yield great investment and losses costs. 2) Using many transformers that result shorter and less expensive conductors.

In general, the area under study is small when compared with the whole distribution network, as well as their loads. Hence, it can be supposed that the part of the primary network within this area will generate insignificant energy losses and voltage drops. Such an assumption permits one to state that the main purpose in primary-transformer interconnection is to install the minimum length of primary feeders.

On the other hand, in the secondary feeder network we should focus our attention on the matters of voltage drops and energy losses due to the low-voltage operation level. The main objective here is to meet the consumers' demand in the shortest path way, which minimizes the energy losses and voltage drops. The secondary conductor size will be chosen according to the flow in each section, and will take

into consideration a set of standard secondary cables. Each cable has an acquisition cost and generates a specific energy loss according to its power flow. The conductor size appropriate for each section will be the one that minimizes a composite function of its losses and investment costs.

3. SECONDARY SYSTEM MODEL

An optimal solution to the secondary network design problem will have to meet the forecast demand within the technical operation standards at a minimum cost. This cost is composed of two parts: investments and losses. They have different temporal features. Investment takes place over a short period of time whereas the cost of losses occurs during all of the equipment's useful life. To overcome this difficulty, the investment cost equivalent annual charge is used [4].

The area selected to design the secondary network can be easily represented as a non-oriented graph G(N,M), where N is the set of nodes and M the set of edges. The set $N = \{1, 2, ..., n\}$ indicates the location of the poles in the area. Each node is assigned a demand relating to the consumers connected to the respective pole. The edges represent the possible links between nodes, and each edge is associated with a given length.

Figure 1 shows the total annual cost per kilometer for a set of standard secondary distribution cables. It is easy to observe that there is an economical cable choice dictated by the power flow ranges. An interesting aspect is that a simple economical analysis is enough to determine which cable must be used, because this factor is more restrictive than the conductor capacity. As shown in Figure 1 the economical

use of the first cable ranges from zero to 11kVA although its power flow limit is 68kVA (220V).

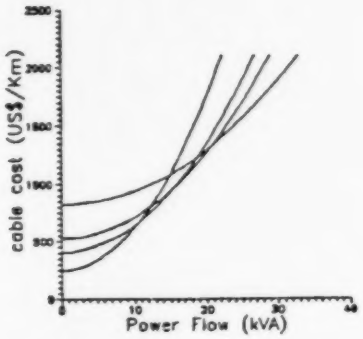

Figure 1 : Annualized feeder cost functions

To avoid defining several edges representing each one of the different cable sizes, the edge cost function can be defined by the inferior linear approximation of the cable cost functions [5]. For the sake of accuracy a fixed charge, representing a feeder fixed cost, may be considered.

With the graph and the primary voltage supply points established, the low-voltage network design problem comprises the transformer location and sizing, the secondary feeder routing and sizing and, finally, the primary-transformer interconnection. The primary interconnection can be made in a cascading fashion, i.e., the same primary branch can be used to connect several transformers. Therefore, the primary feeder routes inside the secondary subsystem can be optimized. As we already discussed in the previous section the objective function in this problem is to minimize the primary network length.

Notice that the paths assigned to primary feeders may be the same as those for secondary feeders, so the resulting graph should be defined as a two level graph. The

upper level contains the primary feeders' possible routes and the lower level those of the secondary ones. Connecting these two levels there are arcs (oriented edges) which represent the possibilites of transformer installation. These arcs join each pair of corresponding nodes, as shown in Figure 2. A transformer size is represented by an arc which has associated the corresponding cost function. For the sake of clarity only one transformer size is used in Figure 2.

There are some differences between the upper level network and the lower level network. The primary subsystem model has only transhipment nodes (zero demand) and the edge cost function refers only to investments. The objective is to minimize the total primary length. In the lower level network model (secondary subsystem) the nodes may have assigned a demand and the edge cost also includes energy losses. Here the objective function is to minimize the total investment and the cost of energy losses.

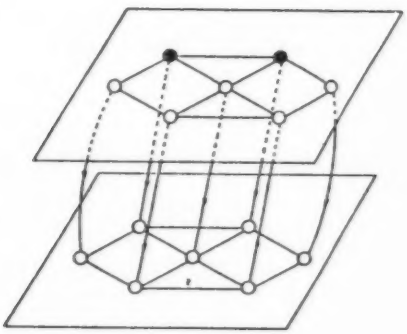

Figure 2 : Two level primary-secondary interconnected network model

The two level design problem can be formulated as a capacited facility location problem with additional radiality and voltage drop constraints. To represent the transformer (or the feeder section) fixed cost we define a binary variable associated with its corresponding arc (or edge). If the zero-one variable is set to unity, the element is installed; otherwise, it is not. The two level problem can be stated as :

$$Min \quad \sum_{j \epsilon M} (\mid c_j x_j \mid + F_j y_j) \tag{1}$$

subject to:

$$Ax = b \tag{2}$$

$$-y_j \overline{x}_j \leq x_j \leq y_j \overline{x}_j \qquad \forall_j \epsilon M_1 \tag{3}$$

$$0 \leq x_j \leq y_j \overline{x}_j \qquad \forall_j \epsilon M_2 \tag{4}$$

$$\sum_{j \epsilon S_k} y_j \leq 1 \qquad \forall_j \epsilon N \tag{5}$$

$$y_j \epsilon \{0, 1\} \qquad \forall_j \epsilon N \tag{6}$$

radiality constraints (7)

voltage drop constraints (8)

where

N	set of nodes
M	set of edges and arcs ($M = M_1 \cup M_2$)
M_1	set of edges in the upper and lower levels
M_2	set of arcs connecting the two levels
S_k	set of edges in M_2 which are associated with pole k
A	node-edge incidence matrix
x_j	power flow in edge j
b	power demand vector
c_j	annualized variable cost for element j
F_j	annualized fixed cost for element j
y_j	binary variable associated with element j
$\overline{x}_j$	power flow upper limit in element j

The objective function (1) seeks to minimize the total annualized cost which is composed of fixed and variable costs. Equations (2) ensure the power flow conservation at the nodes (Kirchhoff's first law) as well as they guarantee that each demand is met. Constratins (3) and (4) allow only existing or opened ($y_j = 1$) elements to have power flow, and guaran'ee that the equipement capacity is respected. In a new feeder section, i.e., a section which does not currently have a conductor, the flow upper limit $\overline{x}_j$ must be set to a high value. Constraints (5) ensure that no more than one transformer will be installed on a pole, and (6) guarantee the solution's integrality. Constraints (7) require a radial configuration to the secondary network. To satisfy (8) none of the nodes can exceed the voltage drop limit.

4. OVERALL SOLUTION METHODOLOGY

It can be anticipated that a global approach to solve problem (1)-(8) will lead to unacceptable running times due to the large number of zero-one variables in the problem. The transformer location problem would be of enormous dimensions, even without considering the variables that would be generated if the primary and secondary network construction were considered.

To overcome this difficulty we have decided to divide the problem, establishing resolution priorities. First, we handle the problem of holding the voltage drop within given limits. This concern requires us to find good transformer locations and secondary feeder routes. Therefore, the delivery of energy to the transformers in the most economical way has a lower priority, and it is related to the definition of primary feeder routes. The secondary network design problem has itself a dimension that would make it practically impossible to solve. Therefore, another problem division was made. The transformer location problem is solved first, then the secondary feeder routing and sizing problems are treated.

The transformer location problem can be modeled as a p-median problem, a classical uncapacitated location problem which attempts for finding the p load centers (medians) of the area. Its mathematical formulation can be stated as follows:

$$Min \quad \sum_{k \epsilon N} \sum_{j \epsilon N} d_{kj} \, b_k \, x_{kj} \tag{9}$$

$$\text{subject to :} \qquad \sum_{j \epsilon N} x_{kj} = 1 \qquad \forall k \epsilon N \qquad (10)$$

$$\sum_{j \epsilon N} x_{jj} = p \qquad (11)$$

$$x_{kj} \leq x_{jj} \qquad \forall k, j \epsilon N \qquad (12)$$

$$x_{kj} \epsilon \{0, 1\} \qquad \forall k, j \epsilon N \qquad (13)$$

where

N	set of nodes
p	number of medians
b_j	power demand for node j
d_{kj}	shortest path value between node k and node j

$$x_{kj} = \begin{cases} 1 & \textit{if node k is assigned to node j} \\ 0 & \textit{otherwise} \end{cases}$$

Notice that the product $d_{kj}b_k$ that appears in the objective function can be viewed as the electrical momentum for assign node k to node j. The objective function then seeks to minimize the total electrical momentum of the network. Constraints (10) require every node to be connected with exactly one median, while (11) establishes that p medians must be chosen. Inequalities (12) ensure that every node must be connected to a node which has already been chosen as a median ($x_{jj} = 1$). Constraints (13) guarantee the solution's integrality.

Putting aside for a moment the need to choose a suitable value for p, the optimal solution of problem (9)-(13) will locate the p tranformers near the subsystem load centers, minimizing the total electrical momentum and, consequently, the voltage drops. We assume that the cost of placing a transformer on a pole is the same for all poles. Having defined the tranformer locations, we must now identify their service area. This will be done by solving the secondary feeder routing problem which selects the consumers who will be assigned to each transformer. This allows us to define the service area and the transformer capacity which is given by the sum of the loads of the consumers that belong to the area. In case the load of the service area exceeds the largest available transfomer either a consumer reassignment or an increase in the number of transformer in the subsystem can be performed.

Turning our attention to the primary network problem (upper level), a suitable approach to design the feeder routes is to model the problem as a Steiner problem. In this problem the objective is to minimize the total length of the network necessary to cover only a subset t of nodes. In our case, this subset is constituted by the nodes with transformers. Any other node will be covered only if it contributes to minimizing the total network length.

We now return to the earlier question about the number p of medians (transformers) in the problem (9)-(13). In fact, p is a parameter which should be optimized. When the number of transformers increases the primary subsystem cost also increases. But we also observe a decrease in the length of the secondary network as well as in the cost of losses. A heuristic approach to find the optimal value for p is to increase

one by one the number of transformers in the area and evaluate the true cost for each configuration.

Figure 3 shows the overall methodology proposed in this article. The initial number of transformers can be determined by: $ko = \left\lceil \frac{\text{total load of the area}}{\text{largest size transformer capacity}} \right\rceil$ where $\lceil . \rceil$ means the greatest integer less or equal to.

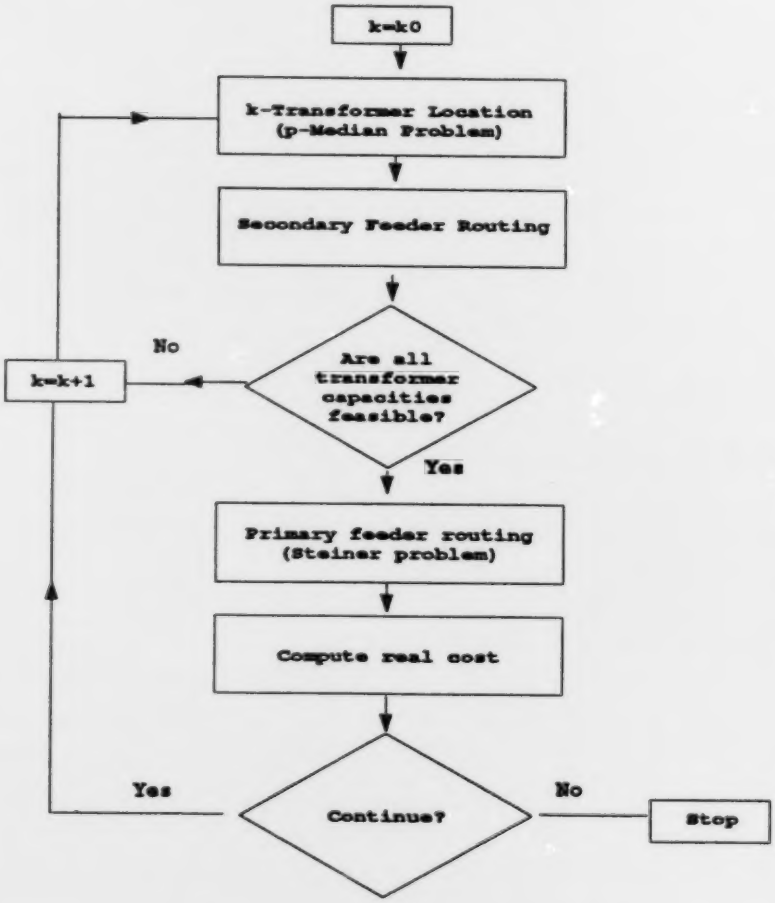

Figure 3 : Flow diagram of the overall methodology

5. SOLUTION TECHNIQUES

In this section we will discuss the solution techniques used to solve each one of the problems presented in the previous section.

Transformer Location

The transformer location problem (9)-(13) has been solved by a "greedy" heuristic procedure which, essentially, seeks to find the best median, then the second best median and so on until the k medians are located. The "greedy" solution can be improved by a node substitution heuristic [6] which has proved to produce near optimal solutions in reasonable computer times. The substitution heuristic tries to improve the incumbent solution by exchanging the current median nodes for other nodes that are not medians. More precisely, it starts with the "greedy" solutions, then chooses a median node i and evaluates the change in the objective function (9) when a non-median node j is substituted for the median node i. If there is an improvement, the set of medians is altered and the process is repeated until no further improvements result.

Secondary Feeder Routing

Having already chosen the sites for the transformers, the next step is to determine the secondary feeder routes and the service areas covered by the transformers. This establishes the power each transformer will supply and also defines the power flow on each feeder section, thus determining the feeder sizes. Consequently, the true cost associated exclusively with the secondary network can be computed.

As shown by objective function (1), the cost function associated with a secondary feeder section is composed of a fixed cost and a (linear) variable cost. If both are considered in the objective function then a branch and bound algorithm could be used in order to obtain an optimal solution, but it would be time consuming. Solutions probably inferior to the optimal solution can be found efficiently if one considers only one of these costs. If the fixed cost is chosen, a minimal spanning tree (MST) algorithm could be used. If, on the other hand, the variable cost is chosen a shortest path algorithm can be used. The MST algorithm is likely to generate very poor solutions because it ignores the distance between transformers and consumers. Therefore, very long routes can result, producing high voltage drops. If a shortest path algorithm is used, we could expect interesting solutions because this procedure implicitly penalizes the losses and the voltage drops. With these matters considered, the solutions obtained must be near to the optimum.

Another fact that supports the use of the shortest path algorithm is that the p-median model used to solve the transformer location problem makes use of the same concept. Remember that the electrical momentum that appears in the objective function of model (9)-(13), employs the shortest path distance d_{kj}. Therefore, initial routes are already available, as well as the load demand, one having to proceed changes only if it happens some infeasibility. For instance, if the load demand within a transformer area exceeds the maximum transformer rating, consumers are transferred from this area to other ones not binding. If this is not possible, increase the number of transformers.

The voltage drop is verified for all nodes in the network. If there is a node that exceeds the allowable limit, we identify the path to the corresponding transformer and a gradual increase in the size of the conductors is attempted, starting from the transformer. If this procedure fails, the current solution is declared unfeasible and the number of transformers is increased by one. It must be emphasized that the aim of the location model is to minimize the electrical momentum which is proportional to the sum of the voltage drops in all nodes. In addition, the cable sizing takes into consideration the power loss as a quadractic function of the power flow. Thus high voltage drops are not expected since they were implicitly considered during the optimization process.

Primary Feeder Routing

The primary-transformer interconnection problem was modelled as a Steiner problem. Real life network models can lead to dimensions which are difficult to solve by exact methods. Therefore, it is necessary again to use a heuristic procedure. Let T be the node set constituted by the primary voltage supply nodes and the nodes where the transformers were assigned. A non-oriented complete graph G' is created where the cost associated to an edge (i,j) is related to the shortest path length between nodes i and j, where $i,j \epsilon T$.

A minimum spanning tree is generated for graph G' and a list containing the edges in the MST solution is created. Starting from this solution, the heuristic procedure attempts to find Steiner points. It is known that to be a Steiner point a node must have at least a degree 3. If a node s has a degree ≥ 3, a list of the shortest path

distances from s to all nodes in T is created and sorted in a non-decreasing order. Next, a merge of this ordered list with the MST solution is performed. The total length of the new MST solution is computed and if a reduction is observed then s is a Steiner point. The procedure tests all Steiner candidate nodes and the node which achieves the greatest reduction in the MST total lenght is inserted into the current MST solution. This procedure is repeated until no Steiner point is found.

6. COMPUTATIONAL TESTS

To test this new methodological approach for the secondary distribution network design problem, a new housing development in the city of Campinas, Brazil was chosen. Figure 4 shows the pole locations (nodes) and all possible routes for the primary and secondary feeders (edges) in the area. This secondary subsystem generates a graph with 142 nodes, 153 edges and 2 primary voltage supply points. The forecast demand for each consumer is 4 kVA and the diversification factor is 0.66. The demand for each pole depends on the number of consumers it serves. There are a total of 251 consumers in this area. Table I shows the transformer sizes available for installation and their respective costs. As can be seen, there is a heavy economy of scale in the transformer cost. The conductors are available in four different sizes and their annualized cost functions are shown in Figure 4.

Figure 4 : Possible transformer sites and feeder routes

Table I– Available Transformer Sizes

Capacity (kVA)	Total cost (US $)	Cost/kVA (US$/kVA)
15	178.8	11.92
30	240.6	8.02
45	276.7	6.15
75	348.9	4.65
112.5	468.3	4.16

The iterative procedure depicted in Figure 3 has bee applied to the problem described above. The initial number of transformers was chosen to be 8. The evolution of the total annualized cost according to the number of transformers is shown in Figure 5. The solutions obtained with 8 and 9 transformers produce voltage drops greater than the established limit, defined as 5%, thus were considered unfeasible.

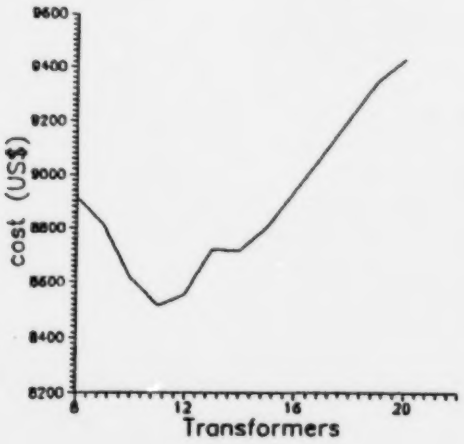

Figure 5 : Total annualized cost

The best solution corresponds to the installation of 11 transformers at a total annualized cost of 8 510. Table II compares three different approaches applied to the same test network. The first one(A) is an empirical method without any mathematical programming recourse which has usually been used to design secondary networks. The second approach(B) uses the secondary and primary routing procedures presented in previous sections but applied to an empirical transformer location solution. The last one(C) is the proposed methodology as shown in Figure 3.

Table II– Cost Comparison

Cost Components	A	B	C
Number of Transf.	12	12	11
Transformers	3 898	3 873	3 632
Sec. Feeders	2 419	2 352	1 599
Losses	1 335	1 356	1 529
Tot. Sec. Cost	7 652	7 581	6 760
Pri. Feeders	1 968	1 877	1 750
TOTAL COST	9 620	9 458	8 510

The total annual cost of the solution found by the proposed method is about 13% less expensive thant the empirical solution, which has a heavier secondary feeder network leading to fewer power losses and more expensive investments. The best solution obtained by approach B is about 11% more expensive than the proposed, showing that the transformer location is a very important and difficult task.

The transformer sites and the secondary feeder routes found by the proposed method are shown in Figure 6. Figure 7 shows the respective solution obtained by

the empirical method. The disconnected points in the graph have no demand. The number that appears next to transformers (triangles) are their maximum capacities.

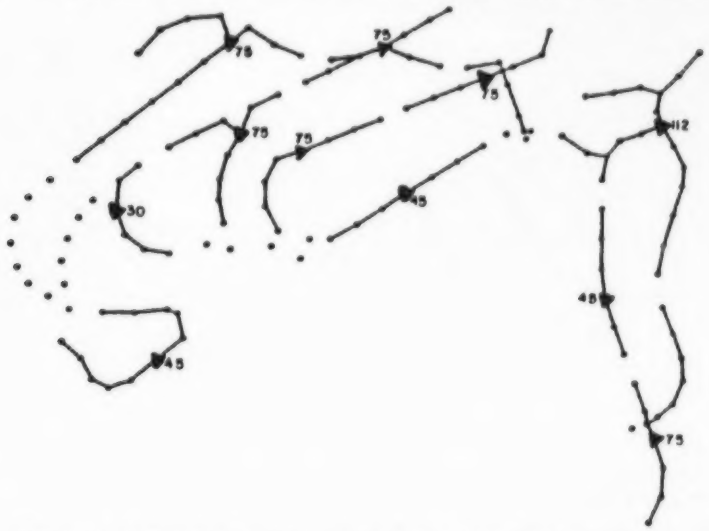

Figure 6 : Secondary transformer-feeder radial solution : proposed method

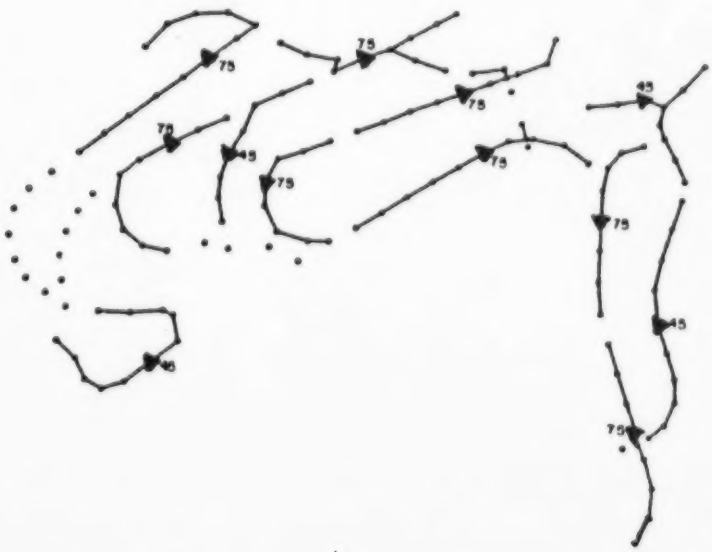

Figure 7 : Secondary transformer-feeder radial solution : empirical method

Figures 8 and 9 respectively show the proposed and empirical solutions for the primary feeder routing problem. They total 2170 and 2437 meters, respectively.

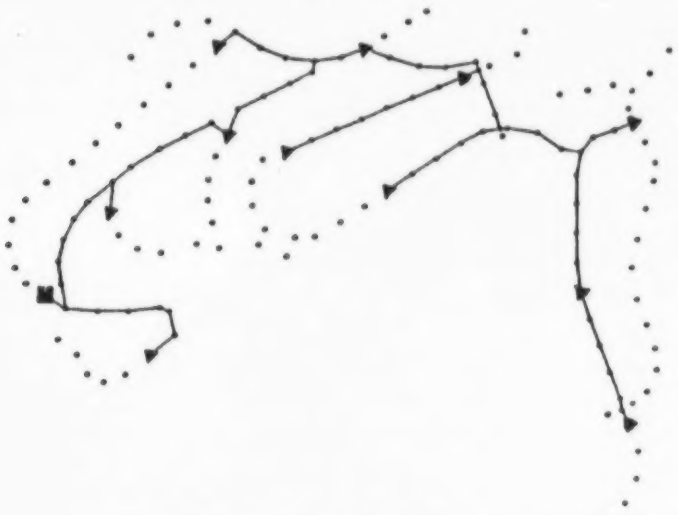

Figure 8 : Primary feeder routes : proposed method

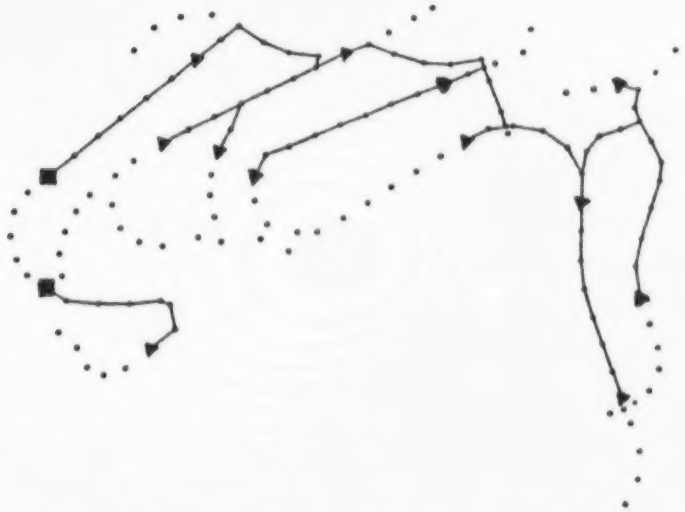

Figure 9 : Primary feeder routes : empirical method

The substitution heuristic used in the transformer location phase has improved the "greedy" solution in all 13 tested configuration, which proves its relevance in the solution process. For instance, in the best secondary solution shown in Figure 6, the objective function value was reduced by about 5.4%. The improvements tend to decrease when the number of transformers increases.

The heuristic used to solve the Steiner problem always reduces the primary feeder total length obtained by the MST algorithm. In the best solution the total length was reduced from 2 499 to 2 176 meters, a saving of about 13%. This number reflects the average savings for every network configuration.

In order to evaluate the best secondary feeder routes achieved by the shortest path approach, we compared this solution with a solution obtained by an exact branch and bound algorithm which takes into account the feeder fixed costs. For all tested transformer location configurations, the branch and bound method yielded optimal solutions practically almost identical to the heuristic solutions.

The programs have been implemented in C language and the tests have been run on a 286 microcomputer with a 10 MHz clock. The average running time to obtain the best configuration for a given number of transformers has been approximately 10 seconds, which enables exhaustive tests and analyses of several demand scenarios.

7. CONCLUSIONS

This article proposes an optimization model for the secondary network design problem which takes into account the investment cost on primary and secondary feeders and transformers, and the cost of power losses. The solution approach divides

the model into three subproblems: transformer location problem, secondary feeder routing problem and primary interconnection problem. The optimization techniques used to solve the subproblem are mainly heuristics procedures which enables the method to be applied to large secondary networks.

Results from several tests carried out on a real instance have proved that this computer-aided secondary network tool can produce solutions that overcome those obtained by the empirical methodology commonly used. The total number of transformers as been shown to affect the quality of the solutions thus justifying the proposed hierarchical decomposition approach.

To extend this methodology to the network expansion problem, the equipment already installed in the area must be considered in the model. We believe that this adjustment can be accomplished without great difficulty. This problem is a subject of the authors' current research.

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